COMPARATIVE ANALYSIS OF THE LEADING CONSUMPTION-BASED ASSET PRICING MODELS

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Absract: The article analyzes in depth the consumption-based asset pricing models, and displays most perspective contemporary trends in the field. A conceptual framework of models has been originally presented linking macroeconomic and financial relationships, and mathematical basis of the classic CCAPM has been developed. The paper also brings out the leading approaches for modification of the basic model, overcoming some of its shortcomings, and analyzes the advantages, disadvantages and the ability of consumption-based modern models to recreate empirical correlations in profitability and the risk of financial assets. The leading conclusion of the article is that there is still no convincing rational consensus model to reproduce adequately the characteristics of financial markets. From an econometric perspective, the closest in this endeavour is the model of long-term risk of Bansal and Yaron (2004) and its modifications.

Key words: consumption-based asset pricing models, recursive preferences, long-run risk, heterogeneous consumers, equity risk premium.

JEL: G12, G14

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Introduction

The real economy and finance are inseparably linked but in the academic literature, with the aim of specialization of knowledge, they are often studied in isolation. This approach embodies largely the core of financial engineering, or asset pricing. The leading traditional financial models such as CAPM, APT, three-factor Fama-French model and others overlook a number of macroeconomic dependencies and sources of risk. Their main focus is on portfolio solutions of investors who take into account only the change in their fortune for a period in the future. The reduction of influencing factors allows these models to have relatively simplified mathematical structure and felicitous form enabling their easy application in investment practice. In the same time, however, it is the reason for their essential constraints beyond the expected compromise in accuracy. The models are unable to show indicators that determine the value of the risk-free return and hence, its fair value, and more importantly - they use factors exogenous to them (e.g. market return or set of fundamental indicators) when bringing out the remuneration for the risk taken by the investors, thus making possible the comparable valuation of an asset based on price dynamics of other assets or risk factors. Therefore, some important problems related to the value of the market returns, its variation, and the dynamics of risk premiums of individual assets cannot be studied. For its part, including macroeconomic dependencies the inter-temporal pricing models allow for exploring systematic risk factors shaping market prices. By marginal rate of substitution of consumption or the so called stochastic discount factor, the systematic risk is included as an endogenous variable in this class of models. Portfolio solutions of the investors and expected profitability are simultaneously bound with their preferences related to current and future consumption. Although created later, this theory should be considered as a basis in the field of asset pricing, because all other asset pricing models can be viewed as special cases of the parent consumption-based asset pricing model. [1]

The limitations of traditional financial models and their generally unsatisfactory empirical results create favorable conditions for the development of models binding finance and macroeconomics. Over the last twenty years, prospects of the consumption-based models have formed a real boom in the development of the asset pricing direction. The abundance of models and approaches to the solution of the financial puzzle involves placing a restrictive parameter of the study. It has several directions: 1) only models relying on rational preferences of investors are subject to analysis; 2) models dealing with valuation of shares, and 3) current models with the highest impact factor undergo selection. Therefore, behavioral patterns, or these for the assessment of bonds and dynamics of the interest rate curve, remain outside the scope of development. It should be noted that studying the success of rational models is in fact, analyzing the possible potential or field for development of models going beyond rationality. While the theoretical basis behind the stochastic discount factor is the same in evaluating bonds and options.

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[1] **Cochrane** Detailed evidence of mathematical relationship is developed by **Cochrane**, J. H. Asset pricing, Vol. 1. 2005

The purpose of the development is to bring out the most promising contemporary trends in the field of asset pricing by using rational consumption-based models. The high dynamics in this area, as well as inadequate use of based consumption models, has formed in the Bulgarian academic literature actuality of the study and determined its implementation.

1. Characteristics of the consumption-based asset pricing model

Trading financial assets can be seen as a cycle of consumption transfer over time - deprivation of current for future consumption and vice versa. The driving force of this process is the fundamental aspiration of the typical economic agent to increase its consumption, ie to raise the standard and quality of life, taking into account the dynamics of the marginal utility of consumption. Thus the preferences of the investors to consumption determine the amounts of financial assets demanded and supplied by them and therefore the equilibrium market prices. Major contributions to the creation of mathematical framework of the basic consumption-based asset pricing model have been made by Rubinstein (1976), Lucas (1978) in a version of continuous compounding, Breeden (1979), for its detailed development and implementation in assessing assets, Grossman and Shiller (1981), Hansen and Jagannathan (1991) and Cochrane (2005) whose works are discussed hereinafter.[2]

In its most simplified form, the choice between current consumption (c_t) and investing in an asset *j*, with price p_t of agent with utility function *u,* is given by:

$$\max_{w} u(c_{g,t}) + E_t\left[\delta\, u(c_{g,t+1})\right], \tag{1}$$

provided that:

$$c_{g,t} = n_{g,t} - p_t w; \; c_{g,t} = n_{g,t+1} + x_{t+1} w,$$

where:

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[2] See. **Rubinstein**, M. The valuation of uncertain income streams and the pricing of options. // The Bell Journal of Economics, 1976, pp. 407-425; **Lucas**, R. E. Asset prices in an exchange economy // Econometrica 46 1978 , pp. 1429–1445; **Breeden**, D. T. An intertemporal asset pricing model with stochastic consumption and investment opportunities. // Journal of financial Economics, 7(3), 1979, pp. 265-296; **Grossman**, S. J. Shiller, R. J. The Determinants of the Variability of Stock Market Prices, // American Economic Review, 71, 1981, pp. 222–227; **Hansen**, L. P., Jagannathan, R. Restrictions on intertemporal marginal rates of substitution implied by asset returns. // Journal of Political Economy, 99(2), 1991, pp. 225-62.

E_0 is the operator of the expected value depending on the information available to the typical agent at time *t*; δ – subjective time discount factor $(0 < \delta < 1)$; n_t – original level of consumption (investor hasn't purchased from asset *j*); w – purchased amount (weight) from asset *j;* x_{t+1} – value (remuneration) of the investment at time *t+1* $(x_{t+1} = p_{t+1} + d_{t+1})$.

According to (1) the investor will decide whether and how to purchase from asset *j,* optimizing their overall utility during the two periods of their life. This decision is affected by the degree of their impatience to consume (δ) and their risk intolerance (γ), as c_{t+1} is an unknown random variable, remote in time. The optimal first-tier application to (2) gives the balance consumption / investment: *u*`

$$(2) \qquad p_t u'(c_{g,t}) = E_t\left\lfloor \delta\, u'(c_{g,t+1}) x_{t+1} \right\rfloor,$$

where:

$u'(c_{g,t})$ – first derivative of increasing, constantly concave function of the utility consumption $u(c_{g,t}) = \frac{1}{1-\gamma} c^{1-\gamma} \Rightarrow u'(c_{g,t}) = c_{g,t}{}^{-\gamma}$.

Equation (2) is an expression of the investment optimum whereby, through purchases or sales of asset *j* , agent g reaches equalization of their marginal costs and marginal benefits. Loss (gain) of the utility from purchase (sale) of an additional unit of the asset during period *t* $(p_t u'(c_{g,t}))$ is accompanied by a corresponding increase (decrease) in discounted utility from the sale of x_{t+1} at time *t+1* $(E_t\left\lfloor \delta\, u'(c_{g,t+1}) x_{t+1} \right\rfloor)$.

By rearrangement of (2), the expected price of an asset *j* can be deduced :

$$(3) \qquad p_t = E_t\left[\delta \frac{u'(c_{g,t+1})}{u'(c_{g,t})} x_{t+1} \right] = E_t\left[M_{g,t+1}\, x_{t+1} \right],$$

where:

$M_{g,t+1} = \delta u'(c_{g,t+1}) / u'(c_{g,t})$- is a marginal rate of substitution or the so called stochastic discount factor - a discount factor which is random (stochastic) as it is not known with certainty at time *t*.

According to the model, the determinants of market price are the expected preferences of investor *g* for the value of their consumption in period *t* and *t + 1*, and the expected value of investment at time *t+1*. (3) is based on the proposition that there is investor g, maximizing the standard feature of utility, that can freely trade the asset *j*. Hence, in the presence of other investors g, whose marginal utility follows different stochastic processes, there are also different $M_{g,t+1}$. However, equation (3) is valid with each individual investor .In specialized literature, significantly greater attention is paid to the generalized version of (3); it is assumed that there are no arbitrage opportunities and transaction costs. Thus the stochastic discount factor is always a positive number and unique value because, trading among themselves, investors eliminate deviations (non-systematic variations) in their marginal utility:

(4) $$p_t = E_t\left[M_{t+1}\, x_{t+1}\right],$$

where:

M_{t+1} is positive and unique (the same for each asset) stochastic discount factor.

If equation (4) is divided by $p_{t,}$ we get:

(5) $$1 = E_t\left[M_{t+1}\, R_{j,t+1}\right] = E_t\left(M_{t+1}\right)E_t\left(R_{j,t+1}\right) + Cov_t\left(M_{t+1}, R_{j,t+1}\right),$$

where:

$R_{j,t+1}$- is the gross return of asset *j* – $R_{j,t+1} = \frac{x_{t+1}}{p_t} = 1 + \Re_{j,t+1}$; Cov_t –covariance dependence.

The expected product from the return of each asset is equal to one, i.e. by (5), M_{t+1} should cover the entire expected return of the asset *j*. This does not mean that all assets have the same return and risk, but that their individual risk characteristics are covered by their covariance with the stochastic discount factor. In this case, the return on the risk-free asset R_{rf} (or a portfolio with zero beta) having $Cov_t\left(M_{t+1}, R_{rf,t+1}\right) = 0$, is equal to:

(6) $$R_{rf,t+1} = \frac{1}{E_t(M_{t+1})}\ .$$

From here, applying (6) in (5), we get:

(7) $$E_t(R_{j,t+1}) - R_{rf,t+1} = -R_{rf,t+1}Cov_t\left(M_{t+1}, R_{j,t+1}\right).$$

The relationship between the return on an asset (its risk premium) and its covariance with the stochastic discount factor is inversely proportional: the lower negative value of $Cov_t\left(M_{t+1}, R_{j,t+1}\right)$, the higher the required rate of return should be. This correlation expresses the sensitivity of investors when presenting the asset j in the individual states of economy. The asset is risky in a period of weak demand and the high marginal utility (eg. in a recession) realizes lower cash flow, and pays off well when investors the least need it - at low utility by further increase in consumption.

The amount of risk premium is limited by variability of the stochastic discount factor. This may be illustrated after rearrangement (7) and transformation of the covariance dependence of its derived variables $(Cov_t\left(M_{t+1}, R_{j,t+1}\right) = \sigma_{(M_{t+1})}\sigma_{(R_{t+1})}\rho_{M,R})$, a $R_{rf,t+1} = 1/E(M_{t+1})$:

$$(8) \qquad \frac{E_t(R_{j,t+1}) - R_{rf,t+1}}{\sigma_{(R_{t+1})}} = -\frac{1}{E(M_{t+1})}\sigma_{(M_{t+1})}\rho_{Rj,M} \quad ,$$

where:

$\sigma_{(M_{t+1})}$ is the standard deviation of M_{t+1}; $\sigma_{(R_{t+1})}$ – the standard deviation of return on the asset j, $R_{j,t+1}$; $\rho_{Rj,M}$ – the correlation between M_{t+1} and $R_{j,t+1}$.

Equation (8) can be further revised, taking into account that mathematically, the correlation $\rho_{Rj,M}$ can range from -1 to 1, allowing (8) to present itself as inequality:

$$(9) \qquad \left|\frac{E_t(R_{j,t+1}) - R_{rf,t+1}}{\sigma_{(R_{t+1})}}\right| \le \frac{\sigma_{(M_{t+1})}}{E(M_{t+1})} \quad .$$

The right side of (9) is the Sharpe ratio of asset j which is limited to the volatility of the unique stochastic discount factor divided by the expected value of M_{t+1} . Equation (9) covers in pure form the relationship between the return and risk of the assets which allows for deducing appropriate restrictions on empirical testing of the model. The correlation $\sigma_{(M_{t+1})}\big/E(M_{t+1})$ determines the slope of the effective front formed by two rays, descended from R_{rf} , on which the assets lie with perfect correlation of returns towards

$M_{t+1}\ (\rho_{Rj,M}=1;-1)$ arranged in ascending order of $\sigma_{(R_{t+1})}$. The most risky assets $(\rho_{Rj,M}=-1)$ are in the upper beam of the effective front, realizing the highest Sharpe ratio. Deviations from perfect correlation indicate the presence of unsystematic risk, which is not rewarded by the market and hence is a cause for the lower return on unit $\sigma_{(R_{t+1})}$.

It is useful to deduce the expected return on asset j also as a function of regression equation (beta evaluation function)[3]:

$$(10)\qquad E_t(R_{j,t+1})=R_{rf,t+1}+\left(\frac{Cov_t(M_{t+1},R_{j,t+1})}{\sigma^2_{(M_{t+1})}}\right)\left(-\frac{\sigma^2_{(M_{t+1})}}{E_t(M_{t+1})}\right)$$

or:

$$(11)\qquad E_t(R_{j,t+1})=R_{rf,t+1}+\beta_{j,m}\lambda_m,$$

where:

$\beta_{j,m}$ is "the amount of consumer risk" or beta of regression equation between $M\ u\ R_j$ measuring systemic risk associated with asset j. λ_m – "cost of risk" depending on the variation of stochastic discount factor. λ_m is the same for all assets, while $\beta_{j,m}$ depends on the individual characteristics of a particular financial instrument.

Equations (10) and (11) are known as consumption capital asset pricing model (CCAPM), because of its resemblance to the CAPM. Here the leading factor in determining the systematic risk ($\beta_{j,m}$) is not the market return but the stochastic discount factor. For the implementation of the model, it is appropriate to replace M_t the marginal utility with the growth of aggregate demand, ie $M_{t+1}=\delta u'(c_{t+1})/u'(c_t)=\delta(C_{t+1}/C_t)^{-\gamma}$. Moreover, this substitution makes it possible to perform linear approximation of $M_{t+1}, M_{t+1}\approx\delta(1-\gamma\Delta\ln C_{t+1}),$ in which (10) takes the form:

$$(12)\qquad E_t(R_{j,t+1})=R_{rf,t+1}+\left(\frac{Cov_t(\Delta\ln C_{t+1},R_{j,t+1})}{\sigma_t^2(\Delta\ln C_{t+1})}\right)\left(\frac{\gamma\sigma_t^2(\Delta\ln C_{t+1})}{1-\gamma\Delta\ln C_{t+1}}\right),$$

where:

[3] **Ludvigson**, S.C. Advances in consumption-based asset pricing: Empirical tests (No. w16810). National Bureau of Economic Research. 2011, p. 7.

ΔlnC_{t+1} is the logarithmic growth of aggregate consumption or consumption per capita.

Unlike traditional financial models, CCAPM is not built from a combination of restrictive and unrealistic assumptions but accepts consumption as a total solution - for investment opportunities, as a measure of the current and future wealth, source of risk, and so on. The profitability of each asset should be the result of its linear relation with consumption and the risk premium, formed by the risk intolerance of typical investor (γ) and the variability of consumption.

2. Modern consumption-based models

From a conceptual point of view, the classic model based on consumption is a comprehensive solution to the problems associated with measuring financial assets and uncertain cash flows. Unfortunately, statistical and empirical tests examining the validity of the model reveal a very different reality. It is unable to re-create historical returns and risk of shares, and the value of relatively risk-free rate of return (short-term government securities). In academic literature these facts have become known as pricing puzzles[4], originally established in the US financial market, and then in other developed markets[5]. Their presence is proved also in the Bulgarian capital market[6]. The object of study in the standard tests of CCAPM is the aggregate market behaviour (leading index for the country) and short-term government securities, the historical values for growth being applied in the formal apparatus, as well as variability of consumption and covariance between consumption growth

[4] These are the puzzle with the risk premium of shares (equity premium puzzle) of Mehra and Prescott (1985) [see also Mehra (2011)], the puzzle with high volatility (volatility puzzle), classified by Campbell (1999) [see also Shiller (1981, 1982)], the puzzle with relatively risk-free return (risk-free rate puzzle), originally identified by Weil (1989). **Mehra**, R., **Prescott**, E. C. The equity premium: A puzzle // Journal of monetary Economics, 15(2), 1985, pp. 145-161; **Mehra**, R. (Ed.). Handbook of the equity risk premium. Elsevier, 2011; **Campbell**, J.Y. Asset prices, consumption and the business cycle, Handbook of Macroeconomics (Elsevier, Amsterdam), 1999, pp. 1231–1303; **Shiller**, R. J. Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? American Economic Review 71(3), 1981, pp. 421–436; **Shiller**, R. J. Consumption, asset markets, and macroeconomic fluctuations. 1982; **Weil**, P. The equity premium puzzle and the risk-free rate puzzle // Journal of Monetary Economics, 24(3), 1989, pp. 401-421.

[5] See. **Campbell**, J. Y. Consumption-based asset pricing. Handbook of the Economics of Finance, 1, 2003, pp. 803-887.

[6] See. **Campbell**, J. Y. Consumption-based asset pricing. Handbook of the Economics of Finance, 1, 2003, pp. 803-887.

and market returns. The results show that in order to generate empirical values of volatility and risk premium of the shares, extremely high levels of relative risk intolerance are required (the risk premium in most countries - over 40, incl. for Bulgaria - $\gamma \approx 52$) [7]. Besides being implausible, the required modelled values of γ lead to excessively high or low levels of relatively risk-free return (the relationship between γ and R_{rf} is not linear and after a certain point, with the increase of γ, R_{rf} starts to drop and passes in negative territory), which calls for assuming, when recreating the empirical values, that the typical investor has a strong reluctance to postpone their consumption (low, unrealistic levels of δ). In addition, the relative risk intolerance is constant in time. This is not supported by empirical data about market volatility, which is largely a result of changes in risk preferences of investors in the various states of the economy. The generated variability of the logarithmic market return in the base model is limited to the flow of dividends. On this basis, a modification of the basic model is required to enable the generation of low risk-free rate of return, higher risk premium and volatility of shares, in the presence the low variability of consumption and covariance of consumption growth with market returns. The directions to achieve these purposes, depending on rational economic agents are basically three: 1) changing the preferences of investors; 2) placing of heterogeneous users; 3) different approaches taking into account the dynamics of consumption. These approaches are consistently subjected to analysis in the following sub-paragraphs.

2.1. Modifications of the preferences of investors (different utility functions)

a) Recursive utility - Epstein-Zin-Weil model

With the standard utility function between the relative risk intolerance *(γ)* and the elasticity of inter-temporal substitution *(ψ)*, a close link exists, expressed in: $\gamma = 1 / \psi$. There is insufficient evidence, however, that the interdependence of these parameters must be really strong, which restricts unduly the flexibility of the base model. [8] Based on the theoretical framework of

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[7] In its purest form the relationship between the relative risk intolerance, risk premium and risk-free return can be expressed by deriving the logarithmic premium and R_{rf} : $lnE(R_e) - lnR_{rf} = \gamma\sigma_{t,\Delta ln(C)} corr[ln(R_{e,t}), ln(\Delta C)]\sigma_{t,\Delta ln(Re)}$;

$$ln R_f = -ln\delta + \gamma\Delta ln(C) - \frac{1}{2}\gamma^2\sigma^2_{t,\Delta ln(C)}.$$

[8] Risk intolerance expresses the willingness of economic agents to prefer reliable results to the insecure ones at the same expected benefit, i.e. to reduce the uncertainty of the change in consumption in different states of the economy. For its part, the elasticity of intertemporal substitution expresses the ratio between the change in consumption and the change

Krebs and Porteus[9] (1978), Epstein and Zin[10] (1989, 1991) and Weil[11](1989) (EZW) developed a model which overcomes that limitation. The function of recursive utility EZW can be represented by[12]:

$$(13)\qquad U_t = \left\{ (1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}},$$

where:

$\theta = (1-\gamma)/(1-1/\psi)$. In a state of $\gamma = 1 / \psi$ $(\theta = 1)$, the recursive utility becomes linear and can be converted to standard utility function.

The budget constraint of the typical investor connecting total wealth (W_t) and consumption (C_t), e equal to:

$$(14)\qquad W_{t+1} = R_{w,t}(W_t - C_t),$$

where:

W_{t+1} is the total wealth for a period t+1; $R_{w,t}$ – gross return of the portfolio, including all invested wealth.

By (14), a proposition is made that in a state of equilibrium the total value of dividends (including wages) is equal to the aggregate consumption. The budget constraint (14) poses a serious obstacle to empirical testing of the model as it is very difficult (rather impossible) to derive returns $R_{w,t}$ t. Thus the use of a felicitous substitute is needed which, according to Epstein and Zin (1991) is the return of the leading market index. From here EZW derived the first- order optimality on the Euler equation as:

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in the utility during a given period, ie the willingness of individuals to substitute consumption over time due to changes in the expected returns or interest rates. The investors' risk preferences are present whether refers to one or multiple periods while the elasticity of inter-temporal substitution exists whether there is uncertainty or not. For more details see **Lybbert**, T. J., McPeak, J. Risk and inter-temporal substitution: Livestock portfolios and off-take among Kenyan pastoralists. // Journal of Development Economics, 97(2), 2012, pp. 415-426.

[9] See. **Kreps**, D. M., Porteus, E. L. Temporal resolution of uncertainty and dynamic choice theory. // Econometrica: Journal of the Econometric Society, 1978, pp. 185-200.

[10] See. **Kreps**, D. M., Porteus, E. L. Temporal resolution of uncertainty and dynamic choice theory. // Econometrica: journal of the Econometric Society, 1978, pp. 185-200.

[11] See. **Epstein**, L. G., Zin, S. E. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. // Econometrica: Journal of the Econometric Society, 1989, pp. 937-969. **Epstein**, L. G., Zin, S. E. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis. // Journal of political Economy, 1991, pp. 263-286.

[12] **Campbell**, J. Y., Lo, A. W. C., MacKinlay, A. C. The econometrics of financial markets. Princeton, NJ: princeton University press. 1997, p. 319.

$$(15) \quad 1 = \lfloor M_{t+1} \quad R_{j,t+1} \rfloor,$$

where:

$$M_{t+1} = \delta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{\frac{-\theta}{\psi}} (R_{w,t+1})^{\theta-1}; \; R_{j,t+1} - \text{expected return on asset}$$

j.

If it is assumed that the return on assets and consumption growth have normal logarithmic distribution, the stochastic discount factor takes the form:

$$(16) \quad m_{t+1} = \theta ln(\delta) - \frac{\theta}{\psi} ln(\frac{C_{t+1}}{C_t}) + (\theta - 1) ln(R_{w,t+1}).$$

Under these conditions, the value of risk-free return with continuous compounding is formed by:

$$(17) \; r_{f,t+1} = -ln(\delta) + \frac{E_t[\Delta c_{t+1}]}{\psi} - \frac{1-\theta}{2} \sigma_w^2 - \frac{\theta}{2\psi^2} \sigma_c^2 \; ,$$

where:

$E_t[\Delta c_{t+1}]$ - is the expected logarithmic growth of consumption; σ_w^2 – the dispersion of logarithmic market return; σ_c^2 – the dispersion of logarithmic growth of consumption.

The impact of the expected consumption growth on the amount of risk-free return is determined by the elasticity of inter-temporal substitution. The higher is the value of ψ (by unit), the less impact the change in consumption will have on the risk-free return, which allows the use of higher levels of relative risk intolerance with the calibration of the model and therefore provides a potential solution to the puzzle with the value of $r_{f,t+1}$. Unlike the base model, this risk-free return is a function of both the protection provided by consumerist risk and the volatility of returns on wealth. Depending on the amount σ_c^2 and σ_w^2 , investors are willing to pay higher price for the risk-free asset. It should be borne in mind, however, that at high values of θ the last two terms of equation (17) can have a positive value.

The logarithmic premium of each risk asset j (including market portfolio) is a result of:

(18)

$$E_t\left[r_{j,t+1}\right]-r_{f,t+1}+\frac{\sigma_j^2}{2}=\frac{\theta}{\psi}Cov_t(r_{j,t+1},\Delta c_{t+1})+(1-\theta)Cov_t(r_{j,t+1},r_{w,t+1}).$$

According to the model of EZW, the risk premium is formed on the amount of covariance of returns on asset *j* with the consumption growth and market returns weighted by θ (and ψ in σ_{jc}). If $\theta = 1$, the base model is present, while $\theta = 0$ ($\gamma = 1$) gives the traditional CAPM.

Therefore, in the rest of situations ($\theta \neq 1;\ 0$) the risk premium on EZW is a combination of risk factors of CCAPM and CAPM.

The model makes possible the direct application of the equations 17 and 18. In $\psi = 1{,}5$, risk intolerance of 3 and descriptive values for the return of the capital market and consumption derived by Pavlov (2015), there is a risk premium for the Bulgarian capital market of 25, 63%. As a reference, for the base model it is only 0.74%. If the risk intolerance is reduced to 2, the premium gets closer to empirical - 14.74% but the relatively risk-free return still remains improbable of minus 60%. Calculations show significant progress, but clearly not all problems are solved.

Recursive utility and long-term risk

On the basis of recursive utility (model EZW), Bansal[13] and Yaron (2004) (BY) modelled variability and consumption and dividend growth, assuming the existence of a small permanent component in their dynamic lines which is predictable. Similarly to discount models for valuation of shares where the fair value is very sensitive even with little change in the projected long-term growth of the cash flow, BY provide evidence that the current, though small changes in the long-term component in the dynamic lines have a big impact on future dynamics of consumption (dividends). Given that investors can identify changes in this component by the new information about the consumption growth and provide for its future dynamics, its variability then is appropriate to have a significant impact on the risk premium of the shares and their volatility. Thus the long-term risk in the dynamic lines is a potential solution to a number of market puzzles.

Bansal and Yaron (2004) derived the processes of growth in consumption, $\Delta c_{t+1} = ln(C_{t+1}/C_t)$, and dividends, $\Delta d_{t+1} = ln(D_{t+1}/D_t)$, as follows:

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[13] **Bansal**, R., Yaron, A. Risks for the long-run: A potential resolution of asset pricing puzzles // Journal of Finance, 59(4), 2004, pp. 1481–1509

$$(19) \quad \begin{aligned} x_t &= \rho x_{t-1} + \varphi\sigma_t e_{t+1} \\ \Delta c_{t+1} &= \mu_c + x_t + \sigma_t \eta_{t+1} \\ \Delta d_{t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1} \\ \sigma_{t+1}^2 &= \sigma^2 + v_1\left(\sigma_t^2 - \sigma^2\right) + \sigma_w w_{t+1} \end{aligned},$$

where:

x_t is the long-term risk in the processes of consumption and dividend growth ; μ_c *и* μ_d are the average growth rates, c_t *и* d_t; e_{t+1}, η_{t+1} and u_{t+1} are shocks with identical and independent distribution (i.i.d.); parameters $\phi > 1$ and $\varphi > 1$ aim at recreating the empirical regularity associated with higher variability of dividends towards consumption (ϕ can be considered as leverage ratio); σ_{t+1}^2 – includes the changing over time economic uncertainty in consumption growth with an average σ^2.

The constancy in the expected growth of consumption and its variability is measured by the parameters ρ ($0 < \rho < 1$) и v_1 ($0 < v_1 < 1$). BY derived empirical evidence that $\rho = 0.979$, а $v_1 = 0.987$, which implies a very slow change over time of x_t и σ_t^2. Hence, the changes in x_t have long-term impact on consumption and dividend growth. For convenience, the shocks e_{t+1}, η_{t+1} and u_{t+1} are modeled as mutually independent. In most pieces of literature related to the long-term risk, the basic equation (19) of BY is developed by adding the component $\phi_c \sigma_t \eta_{t+1}$ in the process Δd_{t+1}. With that, the dividend growth is allowed to be exposed to temporary shocks in consumption. Croce[14], Lettau and Ludvigson (2008) qualify $\sigma_t \eta_{t+1}$ as short-term risk because it increases the systematic risk of the market portfolio due to its connectivity with the stochastic discount factor, but for a short period, because it is *i.i.d.*

BY regarded the aggregate consumption and dividends as two different processes that model consistently[15]. They apply the linear-logarithmic approximation of Campbell[16] and Shiller (1988) for calculation of return on individual assets. The return on an asset allocating dividends identical to the flow of consumption, is approximately equal to:

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[14] **Croce**, M.M., Lettau, M. and Ludvigson, S.C. Investor information, long-run risk, and the duration of risky cash flows. In AFA 2008 New Orleans Meetings., 2008. Available at SSRN: http://ssrn.com/abstract=960886.

[15] Thus economic agents have access to employment income. The current article only presents deriving the income from an asset providing the flow of aggregate consumption (R_w), the mathematical framework for the income from market portfolio.

[16] See. **Campbell**, J. Y., Shiller, R. J. The dividend-price ratio and expectations of future dividends and discount factors. // Review of financial studies, 1(3), 1988, pp. 195-228.

(20) $r_{w,t+1} \approx k_0 + k_1 z_{t+1} + z_t - \Delta c_{t+1}$,

where:

z_t is the logarithmic multiplier price / consumption $(z_t = ln(P_t / C_t))$; $\Delta c_{t+1} = ln(C_{t+1}/C_t)$; k_0 *u* k_1 are constants that depend on the average value of $z_t - k_0 = ln(1+e^{\bar{z}}) - k_1 \bar{z}$, $k_1 = \frac{e^{\bar{z}}}{1+e^{\bar{z}}}$.

The future value of the coefficient price-consumption is given by:

(21) $z_{t+1} = A_0 + A_1 x_{t+1} + A_2 \sigma_{t+1}^2$,

where:

$$A_0 = \frac{ln(\delta) + \mu(1-\frac{1}{\psi}) + k_0 + A_2 k_1 \sigma^2 (1-v_1) + A_2^2 k_1^2 \sigma_w^2 \frac{\theta}{2}}{1-k_1};$$

$$A_1 = \frac{1-\frac{1}{\psi}}{1-\rho k_1};$$

$$A_2 = \frac{0.5\left[\left(\theta - \frac{\theta}{\psi}\right)^2 + \left(\theta A_1 k_1 \varphi_e\right)^2\right]}{\theta(1-k_1 v_1)}.$$

Here, substituting (20) in (16), the logarithmic stochastic discount factor can be deduced[17] :

(22)
$$m_{t+1} = (\theta-1)q + ln(\delta) - \frac{\mu}{\psi} - \frac{x_t}{\psi} + (\theta-1)A_2(k_1 v_1 - 1)\sigma_t^2$$
$$+\left(\theta - 1 - \frac{\theta}{\psi}\right)\sigma_t \eta_{t+1} + (\theta-1)k_1 A_1 \varphi \sigma_t e_{t+1} + (\theta-1)k_1 A_2 \sigma_w w_{t+1}$$

,

where:

$q = ln(\delta) + \mu(1-\frac{1}{\psi}) + k_0 + A_0(k_1 - 1) + A_2 k_1 \sigma^2 (1-v_1)$;

$\theta - 1 - \frac{\theta}{\psi} = \theta(1-\frac{1}{\psi}) - 1 = -\gamma$

[17] **Munk**, C. Financial asset pricing theory. Oxford University Press, 2013, p. 339.

At laying

$E[m_{t+1}] = (\theta - 1)q + ln(\delta) - \frac{\mu}{\psi} - \frac{x_t}{\psi} + (\theta - 1)A_2(k_1 v_1 - 1)\sigma_t^2$,

$\lambda_\eta = \gamma, \lambda_e = (\theta - 1)k_1 A_1 \varphi \; u \; \lambda_w = (\theta - 1)k_1 A_2$ the equation (22) takes the form:

$$(23)\ m_{t+1} = E[m_{t+1}] - \lambda_\eta \sigma_t \eta_{t+1} - \lambda_e \sigma_t e_{t+1} - \lambda_w \sigma_w w_{t+1},$$

where:

$\lambda_{\eta;e;w}$ expresses the market value of the risk of shocks, respectively in consumption, the expected growth of consumption and its variability.

The expected logarithmic risk premium for a hypothetical asset providing the flow of aggregate consumption (*Rw*), is equal to:

$$(24)\ E[r_{w,t+1} - r_{f,t}] = -\lambda_\eta \sigma_t^2 - \lambda_e k_1 A_1 \varphi \sigma_t^2 - \lambda_w k_1 A_2 \sigma_w^2 - 0.5\, var(r_{w,t+1}),$$

where:

$$Var_t(r_{w,t+1}) = \sigma_t^2 + k_1^2 A_1^2 \varphi^2 \sigma_t^2 - k_1^2 A_2^2 \sigma_w^2 .$$

At $\gamma = 10$, $\psi = 1{,}5$ and $\phi = 3$, the model of Bansal and Yaron managed to generate the behaviour of financial markets and risk-free asset, similar to the historical. After a series of quantitative tests, Bansal, Gallant, and Tauchen (2007) also confirmed the validity of the model. The authors develop a methodology allowing for comparable assessment of the capabilities of this model and the habit model (Campbell and Cochrane (1999), discussed in the next subparagraph) and found that the long-run risk model is doing better. Further evidence for long-run risk existence in the growth of consumption and its importance for market returns is provided by Hansen, Heaton and Li (2008) and Bansal, Kiku, and Yaron (2012) [18]. Campbell and Beeler[19] (2009) criticized the model of BY and its modification by Bansal et al. (2007) mainly in two directions: 1) the inability of the generated coefficient price-dividend to predict the future dynamics of the risk premium and at the same time, its overly strong correlation with the future growth of consumption in the long term, which is not supported by the empirical data; 2) crucial dependence of

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[18] Bansal, R., Kiku, D., & Yaron, A. (2012). Risks for the long run: Estimation with time aggregation (No. w18305). National Bureau of Economic Research; **Bansal**, R., Kiku, D., Yaron, A. Risks for the long run: Estimation with time aggregation (No. w18305). National Bureau of Economic Research. 2012.

[19] **Beeler**, J., Campbell, J. Y. The long-run risks model and aggregate asset prices: an empirical assessment (No. w14788). National Bureau of Economic Research. 2009.

the results generated on the value of the elasticity of inter-temporal substitution (ψ). At values less than one the model cannot recreate market return and risk. According to several studies, the value of ψ is less than one.[20]

b) Habit Formation

The second major approach to modify the preferences of investors is by considering the effects of habit formation in consumption proposed by Sundaresan (1989) and Constantinides (1990).[21]The habit reflects the effects of past consumption on the current marginal utility in consumption. In general, modification of the basic utility function is expressed in adding of a (slowly) varying with time benchmark level of consumption or habit (X_t) which is compared with the level of current consumption. Thus the focus shifts from the absolute value of consumption to its change in the short term. There are two main approaches in modelling the habit in the utility function - as a ratio or a difference towards consumption (C_t / X_t or $C_t – X_t$). For its part, the habit can be "internal", whose form is determined by the own consumption of a given agent or "external", depending on historical levels of aggregate consumption. Essential for final results is the determination of the habit change mechanism by the change of aggregate or personal consumption over time. In literature, two alternative assumptions are applied to the problem - gradual change in consumption by setting the appropriate function or a lag of one period of consumption.

When using a function with a coefficient, Abel[22] (1990, 1996), the utility of this agent is obtained as:

$$(25)\ U_t = E_t \sum_{i=0}^{\infty} \delta^i \frac{(C_{t+i} / X_{t+i})^{1-\gamma} - 1}{1-\gamma},$$

where:

X_t - is internal or external habit that takes into account the influence of past consumption on current utility.

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[20] See. **Hall**, R. E. Intertemporal substitution in consumption. // Journal of Political Economy, 96, 1988, pp. 221–273; **Campbell**, J. Y., Mankiw, N. G. Consumption, income and interest rates: Reinterpreting the time series evidence. In NBER Macroeconomics Annual 1989, Volume 4, pp. 185-246. MIT Press.

[21] See. **Sundaresan**, S. M. Intertemporally dependent preferences and the volatility of consumption and wealth. Review of financial Studies, 2(1), 1989 pp. 73-89; **Constantinides**, G. M. Habit formation: A resolution of the equity premium puzzle. Journal of political Economy, 1990, pp. 519-543.

[22] **Abel**, A. Asset prices under habit formation and catching up with the Joneses. American Economic Review Papers and Proceedings 80,1990, pp. 38–42; **Abel**, A.B. Risk premia and term premia in general equilibrium. // Journal of Monetary Economics, 43, 1999, pp. 3−33.

It is a common practice, when using utility function with a ratio, to assume that the habit is external, which considerably facilitates calculations and modelling capabilities in an environment of normal logarithmic distribution. Stochastic discount factor takes the form:

$$(26)\ M_{t+1} = \delta\left(\frac{X_{t+1}}{X_t}\right)^{1-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

If it is assumed that the habit is formed as a lag of consumption $X_t = C_{t-1}^k$, where κ determines the degree of time-inseparability with a constraint $k(\gamma-1)\geq 0$, (26) is transformed into:

$$(27)\ M_{t+1} = \delta\left(\frac{C_{t-1}}{C_t}\right)^{k(\gamma-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

The researchers in the field of measurement of assets pay significantly more attention to modelling the utility by the difference $C_t - X_t$: Sundaresan (1989), Constantinides (1990), Campbell and Cochrane (1999), Boldrin, Christiano and Fisher (2001), etc.. This interest is prompted by the fact that with the coefficient models of habit formation the risk intolerance is the constant, while the models with a difference have actually changing with the time risk intolerance (time-varying risk aversion), whereby the ability to recreate the empirical data significantly increases. Depending on the state of the economy, the effective risk tolerance decreases at economic growth, while recession grows. The identical agents maximize:

$$(28)\ U_t = E_t\left[\sum_{i=0}^{\infty}\delta^i\frac{(C_{t+i}-X_{t+i})^{1-\gamma}-1}{1-\gamma}\right],$$

where:

X_t is external or internal habit which is expected to be smaller than the consumption, and greater than zero.

With internal habit $M_{t+1} = \delta\frac{MU_{t+1}}{MU_t}$, where marginal utility equals:

$$(29)\ MU_t = (C_t - X_t)^{-\gamma} - E_t\left[\sum_{i=0}^{L}\delta^i(C_{t+i}-X_{t+i})^{-\gamma}\frac{\partial X_{t+i}}{\partial C_t}\right].$$

If an external habit is projected, (29) will be considerably simplified to:

(30) $MU_t = (C_t - X_t)^{-\gamma}$.

One of the most successful models with a difference using the external habit is Campbell[23] and Cochrane's (1999) (CC), whose mechanism will be considered in depth below. CC define the processes of logarithmic consumption and dividend growth as a random walk with a дрифт $\mu_{c;d}$ and a lack of perfect correlation between them:

$$(31)\quad \begin{aligned} &\Delta c_{t+1} = \mu_c + v_{t+1} \\ &\Delta d_{t+1} = \mu_d + w_{t+1} \\ &v_{t+1}, w_{t+1} \sim i.i.d. \quad N(0, \sigma^2). \end{aligned}$$

From the utility function (28) is visible, that the difference $C_{t+1} - X_{t+1}$, or the excess over habit is of particular importance for the model. In this regard, it is helpful to use a coefficient to measure its relative amount towards the consumption (S):

$$(32)\qquad S_t = \frac{C_t - X_t}{C_t}.$$

CC specified the process of changing the logarithmic value of the coefficient $ln S_t \equiv s_t$, and hence, of $ln X_{t+1} \equiv x_{t+1}$ towards the change of consumption:

$$(33)\quad s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(c_{t+1} - c_t - \mu_c),$$

where:

$\bar{s} = ln(\bar{S})$ is the stable level of coefficient s; ϕ controls the constancy of s; $\lambda(s_t)$ is classified as a "sensitivity function" of s_{t+1} and x_t by the changes in consumption:

$$(34)\quad \lambda(s_t) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1 - 2(s_t - \bar{s})} - 1 & s_t \le s_{max} \\ 0 & s_t > s_{max} \end{cases},$$

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[23] **Campbell**, J. Y., Cochrane, J. H. By force of habit: A consumption-based explanation of aggregate stock market behavior. // Journal of Political Economy, 107, 1999, pp. 205–251.

where:

$$\overline{S} = \sigma\sqrt{\gamma/1-\phi}\,;\; s_{max} = \overline{s} + \tfrac{1}{2}(1-\overline{S}^2\,).$$

The process (33) with function (34) ensures that the requirement Ct> Xt will be met, the risk-free return variability is controlled (within these parameters it is eliminated, ie, $r_{f,t}$ is constant), the habit is defined in the region of the stable level of $s\ (\overline{s} = s\,)$ and is slowly changing in parallel with consumption.

It follows from (30) that the stochastic discount factor on the model is equal to:

$$(35)\qquad M_{t+1} = \delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

From equation (35) it is apparent that the classical stochastic factor is developed by adding the ratio S_{t+1}/S_t, whose further variability increases that of M_{t+1} , and hence the generated market volatility and risk premium. It is important to note that γ in this model is not the standard coefficient of relative risk intolerance (γ_{CRRA}), and a parameter of the utility curve, which can be equated by: $\gamma_{CRRA} = \gamma/S_t$. With deterioration of the economic situation (bringing consumption near habit), γCRRA and marginal utility of consumption increase and vice versa, ie by S_t, the counter-cyclical behavior of the risk premium is achieved. The recession of S_t from its average value has a dual opposite effect on interest rates. At low S_t, marginal utility grows and stimulates investors to borrow (knowing that St will return to $\overline{S}$); at the same time, by the rising γ_{CRRA} they would increase their savings. CC assume that the effects are of the same intensity and compensate for each other. Hence, as noted, r_f is a constant:

$$(36)\quad ln\, R_f = -ln\,\delta + \gamma\mu_c - \frac{\sigma^2}{2}\left(\frac{\gamma}{\overline{S}}\right)^2.$$

The difference between (36) and r_f with the traditional CCAPM is in the second member $-\gamma\mu_c$. Here, instead of γ_{CRRA} γ (with its equivalent γ/S_t), the parameter γ is used. This allows, for the same value of γ_{CRRA} to achieve a lower r_f in a more acceptable value of δ. In addition, there is a weakening of the relationship between the average consumption growth and r_f. CC deduced the market return and its variability by coefficient price / consumption (dividend):

$$(37)\quad \frac{P_t}{C_t}(s_t) = E_t\left[M_{t+1}\frac{C_{t+1}}{C_t}\left(1+\frac{P_{t+1}}{C_{t+1}}(s_{t+1})\right)\right].$$

2.2. Models with heterogeneous users

In the majority of financial models it is presumed that there are identical economic agents whose behaviour forms market prices. In fact, it is not necessary for the agents to be identical, but rather the approbation preferences correspond with the specifics of the typical (average) investor because the modelled values such as consumption, interest rates, market rates, etc. are modular, ie result of the influence of all agents and even if there are different classes of market participants, their individual influence blurs and reaches an intermediate level. Questions remain whether the specified "typical" investor is representative for each period or structure and the influence of different classes of agents changes over time and to what extend these aggregation functions are appropriate. It is also questionable whether it is reasonable to aggregate the individual specifics of investors in case of incomplete markets.

A logical starting point in the classification of customers is their division according to the criterion "participation in the capital markets". Well-known is the fact that even in some of the most developed capital markets such as the US, a relatively small proportion of households (48.8% in 2013) own directly or indirectly shares traded. [24] Mankiw[25] and Zeldes (1991) studied the differences in the consumption of both groups of households and their relationship with the dynamics of the equity risk premium. The data show that the consumption of families holding shares has significantly higher volatility and correlation dependence on the equity risk premium. Hence, using only their consumption in the classic CCAPM, the required coefficient of γ for recreating the risk premium is significantly reduced, which makes for explanation (but does not solve completely the puzzle). In this regard, Basak[26] and Cuoco (1998) created a model in which part of the agents cannot participate in the stock market, but only in the market of risk-free asset. The other investors take the entire risk of aggregate consumption and capital market in their flow of consumption. Guo[27](2004) extended the range of modeled variables adding to the limited market participation also uninsurable income risk and borrowing constraints of the individual classes of agents. Representative agents from both classes (i = 1, 2) receive stochastic labor income ($L_{j,1;2}$) at limited L_t =

[24] According to data from Survey of Consumer Finances, there is a tendency for this share to increase - from 32.7 percent in 1989 to its peak of 53.2 percent in 2007. Source: http://www.federalreserve.gov/econresdata/scf/scfindex.htm.

[25] See. Mankiw, N. G., Zeldes, S. P. The consumption of stockholders and nonstockholders. // Journal of financial Economics, 29(1), 1991, pp. 97-112.

[26] **Basak**, S., Cuoco, D. An equilibrium model with restricted stock market participation. // Review of Financial Studies, 11(2), 1998, pp. 309-341.

[27] **Guo**, H. Limited stock market participation and asset prices in a dynamic economy. // Journal of Financial and Quantitative Analysis, 39(03), 2004, pp. 495-516.

$L_{1,t}$ +$L_{2,t}$ and total income in the economy $Y_t = L_t + D_t$. The agents lend and borrow each other through the discount security at the limit of borrowing $\overline{B}_{i,t}$ (always negative) and debt to agent i, $B_{i,t} \geq \overline{B}_{i,t}$, at $B_{1,t} + B_{2,t} = 0$. In the model of Guo (2004) agents maximize traditional function to the utility of (1) with a budget constraint to an agent 1 and 2, respectively: $P_t B_{1,t+1} + P^s{}_t S^1{}_{t+1} + C_{1,t} \leq B_{1,t} + P^s{}_t S^1{}_t + L_{1,t} + D_{1,t}$ и $P_t B_{2,t+1} + C_{2,t} \leq B_{2,t} + L_{2,t}$, where P_t is the equilibrium price of discount security; $P^s{}_t$ - stock price at time t; $S^1{}_{t+1} = S^1{}_t$ amount of shares owned; $C_{i,t}$ - consumption of agent i at time t.

Logarithmic risk-free rate of return for the two classes of agents (i = 1, 2) is deduced in the same way as the classic ($rf_{i,t+1} = -ln\,\delta + \gamma E[\Delta ln(C_i)|\Omega_t] - \frac{1}{2}\gamma^2\sigma^2_{t+1,\Delta ln(Ci)}$), , and the equilibrium rf results from their minimum:

$$(38)\quad rf_{t+1} = min(rf_{1,t+1}, rf_{2,t+1}).$$

Logarithmic risk premium on the model of Guo, is equal to:

$$(39)\quad E[r_{t+1} - rf_{t+1}|\Omega_t] + \frac{\sigma^2_{r,t+1}}{2} = \gamma\, cov_{t+1}(r_t, c_{1,t}) + rf_{1,t+1} - min(rf_{1,t+1}, rf_{2,t+1}),$$

where:

$cov_{t+1}(r_t, c_{1,t})$ is the covariance between market returns and the shareholder consumption growth; $rf_{1,t+1} - min(rf_{1,t+1}, rf_{2,t+1})$ - "liquidity premium" for holding of shares; E [.] - operator of the expected premium at current information Ω_t..

The first member in (39), $\gamma cov_{t+1}(r_t, c_{1,t})$ is the premium on the classic CCAPM, but taking into account the effect of the limited market participation - the aggregate consumption has been replaced by more volatile shareholder consumption bound to the market return (agent 1). The second member is the result of restrictions with indebtedness (without them $rf_{1,t+1} = rf_{2,t+1}$) and can be described as a liquidity premium, because the investor in stocks cannot use them as a protection from shocks in consumption. The sum of the two members significantly increases the expected risk premium without needing high levels of γ. At the same time, the higher variability of c_1 does not increase automatically variability and risk-free asset return, as rf_{t+1} is a result of marginal propensity to substitute both agents in the economy. Balance in the economy is available when: $C_{1,t} + C_{2,t} = L_{1,t} + L_{2,t} + D_{1,t}$.

Another noteworthy approach in modeling the heterogeneity of investors, is offered by Brav, Constantinides and Geczy (2002). They draw the attention from the aggregate consumption per capita to the consumption of individual households ($c_{i,t}$). The authors found that the stochastic discount factor of individual households is valid and hence their weighted average:

$$(40)\ M_{t+1} = \delta\left(\frac{1}{N}\sum_{i=1}^{N}\left(\frac{c_{i,t}}{c_{i,t-1}}\right)^{-\gamma}\right).$$

According to Brav et al. (2002), the average discount factor of households shows enough variability to generate empirical risk premium at a low value of γ. In comparison, if the standard M_{t+1} is applied with the dynamics of agregated household consumption, the results will be considerably worse, indicating that markets have not been completed (complete markets). In this regard, Brav et al. (2002) performed additional tests, in which they use dynamics in consumption only to those households holding shares with a market value above a certain threshold. By increasing the threshold, the required γ decreases, which can be interpreted as evidence that major and active investors largely determine market return. However, the required γ, when using the highest threshold of $ 40,000 is significantly higher than that in (40).

2.3. Different approaches to consider the dynamics of consumption

Other hypotheses worthy of attention about the increase of variability of the stochastic discount factor and hence, the equity risk premium suggest: adding a premium for rare disasters and the probability of survival in the market. Rietz[28] (1988) revised the traditionally applied approach of normal distribution of logarithmic consumption growth and proposed to analyze the effect on the expected return by large (catastrophic) decrease in consumption. If investors expect that, even very rare, these catastrophic events to happen, they will rationally increase the cost of risk-free asset and will require higher returns from the risk assets. Therefore, the critical parameters in the model of Rietz are the probability of such an event and its magnitude. According to Rietz, at acceptable values of these parameters (1% probability of a 25% drop in consumption), the premium of shares can be recreated with γ from 10. Criticism about the validity of Rietz's assumptions motivated Barro[29] (2006) to pay attention to the calibration of the model through a thorough analysis of international catastrophic events in history. He estimated that there are 1.5-2% probability of annual decline in GDP per capita between 15% and 64%. The

[28] **Rietz**, T. A. The equity risk premium: A solution. // Journal of Monetary Economics 22, 1988, pp. 117–131.

[29] See. **Barro**, R. J. Rare disasters and asset markets in the twentieth century. The Quarterly Journal of Economics, 2006, pp. 823-866.

Barro model manages to reach empirical premium, combining the most extreme historical values and $\gamma = 4$. A commonly encountered note in the academic literature on these results relates to the fact that the quoted rates of drastic decline are the accumulated values for several consecutive years (most often 3 or 4), while profitability is considered on an annual basis. When aligned to comparable basis, the necessary relative risk intolerance increases significally. Thorough criticism on assumed process of consumption growth and the size of the premium for catastrophic events were presented by Backus[30], Chernov and Martin (2011). By the prices of index options and macro-financial model, the authors derived lower probabilities than Barro of occurrence of extreme shocks in consumption. Julliard[31] and Ghosh (2012) also rejected the proposals of the model based on the modification of the classic CCAPM.

Important development of the models involving catastrophic events was made by Wachter [32] (2013), through the incorporation of changing with the time probability of catastrophe in a recursive utility function. On this basis, Nowotny[33] (2011) added the specification: in case of a serious decline in consumption the probability increases further shocks in the future, similar to those seen in history. Thus, although the likelihood and size of individual shocks are relatively small, the premium can be explained due to changes in investors' expectations of future shocks.

Brown[34], Goetzmann and Ross (1995) offer a concept similar to that of Rietz - a survivorship bias on the market. The high risk premium in many markets is available because they have survived for a long period, unlike the others, for example, in the early twentieth century (Russian, Chinese, Austro-Hungarian, Slovak and so on). From here, the total historical risk premium is supposed to be lower, and the current prelium to reflect investors' expectations about a given market to not survive. Li[35]and Xu (2002) found that for the effect of survivorship bias on the risk premium to be sufficient, the expected (ex-ante) probability assessment of a market must be unrealistically

[30] See. **Backus**, D., Chernov, M., Martin, I. Disasters implied by equity index options. // The journal of finance, 66(6), 2011, pp. 1969-2012.

[31] See. **Julliard**, C., Ghosh, A. Can rare events explain the equity premium puzzle?. // Review of Financial Studies, 25(10), 2012, pp. 3037-3076.

[32] See. **Wachter**, J. A. Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?. // The Journal of Finance, 68(3), 2013, pp. 987-1035.

[33]See. **Nowotny**, M. C. Disaster begets crisis: The role of contagion in financial markets. 2011, Available at SSRN 1714754.

[34] See. **Brown**, S. J., Goetzmann, W. N., Ross, S. A. Survival. // The Journal of Finance, 50(3), 1995, pp. 853-873.

[35] See. **Li**, H., Xu, Y. Survival bias and the equity premium puzzle. // The Journal of Finance, 57(5), 2002, pp. 1981-1995.

low. Another problem with this hypothesis is associated with the necessity for stocks and bonds not to be affected differently in a similar event. History shows, however, that with bonds, significant losses are also observed. After shares and bonds undergo substantial (complete) loss in such scenarios, the difference in return shouldn't be caused by survivorship bias, because investors would require equal or proportionate premium for both instruments.

Conclusion

Over the last twenty years there have been a very dynamic development in the field of consumption-based pricing asset models. The article is an attempt to systematize, amidst the vast universe of alternative models, the leading trends and approaches to explain market behaviour. Here the following conclusions can be derived[36]:

First. According to the classical CCAPM, profitability of each asset should be the result of its linear relation with consumption (consumerism beta) and a risk premium depending on the risk intolerance of typical investor (γ) and the variability of consumption. The model proves to be unable to recreate profitability, risk and predictability in markets, which requires its modification towards a rise in the volatility of the stochastic discount factor and a weakening of the necessary relationship between the dynamics of consumption and return on assets; Second. The combination of recursive utility and the concept for the existence of long-run risk in consumption growth is proving particularly successful. However, while quantitative research favours capabilities of the model it should be considered that the question whether the elasticity of intertemporal substitution (ψ) is higher than one remains open. Habit-based models do well, concerning the reproduction of the empirical premium and risk, and the disadvantages of inherent correlation between consumption growth and return on assets (ratio of price - dividend) which is not typical for the historical data. Models of heterogeneous investors have significant potential as they cover a wide range of variables (real market restrictions), but the results of some studies are controversial and require additional testing. With models involving catastrophic events and survivorship bias, it seems that their basic assumptions have been consolidated as questionable. Third. There is a lack of consensus rational model that recreates with sufficient precision the basic characteristics of financial markets and has commonly agreed assumptions. In order to overcome the shortcomings of

[36] The allocation of copyright participation in the article is as follows: Assoc. Prof. Stoyan Prodanov Ph.D wrote the conclusion, and the rest was prepared by Assist. Tsvetan Pavlov Ph.D

CCAPM, the number of assumptions about the cognitive and professional abilities of the typical investor is increasing, which reduces their realism.

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