THE ANTIMATHEMATICALITY OF DEMAND CURVES[1]

By William Barnett II[*] and Walter Block[*]

Abstract:

Mathematics has proven so helpful in the physical sciences such as physics that it has been improperly applied to economics. The dismal science studies purposeful human action, while purpose in the hard sciences is properly dismissed as anthropomorphic. The present paper takes the demand curve as a case in point. It demonstrates that this tool of analysis is fundamentally flawed in that it violates its own economic assumption of ceteris paribus, and also the mathematical requirement that only the proper number of variables may vary along a given graph in two-dimensional space.

Key Words:

Demand curves, consumer behavior, mathematical economics, marginal rate of substitution, indifference, utility

JEL Category: C6

I. Introduction

The present paper is a contribution to the literature claiming that mathematics is an improper tool in economics.[2]   In section II it is contended that demand curves[3] are antimathematical in that they violate the laws of mathematics. Section III is devoted to the incompatibility between demand curve analysis and singularism. In IV we deal with several other problems with demand curves. We conclude in section V.

II. Mathematical law violation

Standard economic theory maintains that an individual's utility, aka want satisfaction, depends solely upon on the quantities of the n goods he consumes. His implicit utility function is: $U = f(X_i)$ $(i = 1,\ldots, n)$, where the U is his utility and the X_i

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[2] For specifically Austrian criticisms of mathematics in economics, see Anderson, 2001, 2002; Barnett, 2003, 2004; Herbener, 1996; Mises, 1977, 1998; Leoni and Frola, 1977; Rothbard, 1988, 2004; Shostak, 2002; Wutscher, unpublished.

[3] The words "curve(s)" and "graph(s)" are used interchangeably throughout to refer to a graph in two-dimensional, Cartesian space, of a function of two or more variables.

William Barnett II and Walter Block

are the quantities of the n goods. For the sake of simplicity of exposition,[4] n = 2 hereafter, with $X_1 = X$ and $X_2 = Y$. His implicit utility function is, then: $U = f(X, Y)$. That is, the individual's utility is solely dependent upon the quantities of X and Y he consumes.

However, this is unsatisfactory from the point of view of economics, as it leaves no room for preferences, aka tastes. Therefore, any given combination of *X* and *Y* would unvaryingly provide the same level of want satisfaction to an individual. But in the real world individual's preferences change.[5] There must, then, be at least one other variable that affects utility, and that must be the individual's preferences. Note that although utility depends, inter alia, upon preferences, they are different things. Rewriting the utility function with this in mind yields: $U = f(X, Y, T)$, where *T* stands for preferences.

Consider, next, demand. Orthodox theory maintains that a consumer's demand, Q_X, for a good, *X*, is a function[6,7] of the price of *X*, the prices of (at least some) other goods, his income or wealth,[8] and his preferences. In the two-good case as per above, the implicit demand function for X is written: $Q_X = g(P_X, P_Y, W, T)$, where P_X, and P_Y, stand for the prices of *X* and *Y*, respectively, *W* stands for the individual's wealth, and *T* for his preferences.

This brings us to demand curves as graphs of demand functions. A function of three or more variables may be graphed in a 2-dimensional, Cartesian space by measuring the dependent variable on one axis, normally the vertical, and one of the independent variables on the other axis, normally the horizontal. The remaining independent variables are parameters; that is, they are treated as constants when the graph is drawn, so that at every point on the graph the value of each one is constant. As required by the analysis, one or more of the parameters may take on new values. The function may then be re-graphed with the new values of the parameter(s) held constant. Such changes are said to shift the graph, although if there are two or more

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[4] This simplification does not affect the analysis in any substantial way.

[5] For an alternative view on this, one claiming that tastes do not change, see Stigler and Becker (1977). But see also West and McKee (1983).

[6] Because utility functions are used to derive demand curves, and demand curves are concerned with the demand for goods, not the consumption of goods, the only way the derivation of demand curves from utility functions can be valid is if all goods are consumed "immediately" upon purchase thereof. This might seem a pedantic point, however, given the ubiquity and importance of durable consumer goods, due consideration leads to the conclusion that this is not so.

[7] It should be noted that if an individual's demand curve is of the type referred to as a step function, the demand curve is not a graph of a demand function. This oddity arises because of economists' penchant, when drawing supply and demand curves, for measuring the dependent and independent variables on the horizontal and vertical axes, respectively, in exactly the opposite fashion from the standard practice in mathematics. The exact same set of lines in the x-y plane that constitute the graph of a step function y = *f*(x), do *not* constitute an graph of a step function x = *f*(y), because, although there is a one-to-one mapping from *x* to *y*, the mapping from *y* to *x* is not one-to one. In terms of demand curves, this means that the soi-disant step (demand) function is not a function at all.

[8] There is a one-to-one correspondence between wealth and income as income is but the flow of services from wealth and wealth but the appropriately-capitalized value of those services.

William Barnett II and Walter Block

parameters that can alter, it is possible the changes could offset each other with the result that the original demand curve would not shift.

In the case under consideration, the price of X is measured on the vertical axis and (the quantity of) X is measured along the horizontal. W and T are parameters. That is, along any one graph of the demand function, both the individual's wealth and his preferences are held constant. The graph of the demand function is negatively sloped; i.e., $\partial X/\partial P_X < 0$.[9] A change in either W or T, or in both, shifts the graph, save in the latter case if the alterations exactly offset each other.

The standard interpretation of this phenomenon is that, with wealth and preferences constant, the price of X has to decline in order to induce the individual to be willing and able to increase his purchases of X; i.e., to increase his quantity demanded. Now, were this a matter of pure mathematics, it would be of no concern. In fact, however, this concerns economics, not mathematics. It pertains to the purposeful behavior of individual human beings. So, the orthodox economists ask:[10] "Why is it necessary to lower the price in order to induce this person to buy more?"

They offer two different but not completely unrelated answers. The first is in terms of utility, and it is usually articulated along the following lines:[11] It is necessary to lower the price because each unit of the good acquired is put to the most highly valued use the purchaser has for it; therefore, each subsequent unit acquired is of less value than the previous one, and, consequently, the purchaser is not willing to pay as much for it. That is another way of saying that the individual's marginal utility (MU)[12] is different at every point on the demand graph. This at least is the explanation of those economists who seriously adhere to the principle of diminishing marginal utility.

But there is a second answer as well, this one couched in terms of the marginal rate of substitution (MRS) and indifference. The advocates of the soi-disant modern theory of consumer behavior eschew the first, or utility approach, adhering instead to the indifference (curve) analysis. They would say: As an individual acquires additional units of a good, the MRS[13] between that good and any other decreases. That means that as an individual acquires more units of a particular good, he is willing

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[9] Because of economists' transposition of the standard, mathematical order of the axes, the actual slope is the reciprocal of $\partial X/\partial P_X$; however, that is of no matter insofar as the sign is concerned.

[10] Well, they *should* ask.

[11] These two answers are based on any number of microeconomics texts, the repository of learning in the economics profession. This mode of analysis is so prevalent that we need not cite any particular instance of it.

[12] For those who think utility may be quantified, $MU_X = \partial U/\partial X$. For a critique of this viewpoint see Barnett (2003).

[13] The MRS_{YX} is the rate at which the individual consumer would be willing to give up one good for another such that his utility would remain constant. It is defined either as the slope, or the negative of the slope, depending on the definer. The former definition, then, is: $dY/dX_{\text{(for U = constant)}} = -(\partial U/\partial X)/(\partial U/\partial Y)$. On the invalidity of indifference curve analysis, see Barnett (2003), Block (1999, 2003), Hulsmann (1999) and Rothbard (2004).

William Barnett II and Walter Block

to increase the amount of that good he would give up in order to get another unit of some other good; i.e., additional units of a particular good are worth less relative to other goods, ceteris paribus. Therefore, he is not willing to pay as much for the additional unit as he would for the previous one. That is another way of saying that the individual's MRS is different at every point on the demand graph.

But both possible answers violate the laws of economics and mathematics. In the former case, utility varies along demand curves as marginal utility[14] diminishes while quantity increases. This violates the economic assumption of ceteris paribus. Presumably, along a demand curve, the *only* things that are supposed to vary are price and quantity, the elements measured along the vertical and horizontal axes, respectively. If anything else alters, such as income, or tastes, or the prices of complements or substitutes, the demand curve is supposed to *shift.* But here we find that *utility* is changing, as we move along the demand curve. This constitutes a contradiction of the presuppositions of the demand curve. Further, this phenomenon is also incompatible with laws of mathematics as to how many variables may vary along a given graph in two-dimensional space. There is supposed to be a limit of two to a customer; in this case, price and quantity, for the demand curve. The change in utility plays the role of a queue jumper, or an uninvited guest.

In the latter case, the marginal rate of substitution varies along the demand curve, diminishing as quantity increases. This, also, violates the economic assumption of ceteris paribus and the laws of mathematics as to how many variables may vary along a given graph in two-dimensional space. Therefore, all consumer demand curves are antimathematical.

What is the cause of this problem? It is quite simply that the methods appropriate for mathematics and physics, chemistry, etc., are not appropriate for economics. The numbers and symbols of pure mathematics do not have purposes; i.e., they are not the result of their own purposeful actions. When numbers are operated on, it is not because they *choose* to operate on themselves. For example, in the expression 7+1, neither has seven chosen to join with one in a particular fashion, nor has one so chosen in respect of seven. Similarly for physics, the phenomena that are the objects of study of physics are base quantities and quantities that can be derived therefrom (derived quantities); i.e., phenomena that can be *fully* described using the seven SI base units and units derived therefrom (derived units). These quantities do not have purposes; i.e., they are not the result of their own purposeful action. When a particle moves at some fraction of the speed of light, it is not because the particle so chose. For example, in the equation $e = mc^2$, e did not choose to be equated with m and c in any fashion at all, much less as the product of the former and the square of the latter.

However, such is not the case with economics; the phenomena it studies are, precisely, the purposeful actions of human beings (Mises, 1998, Rothbard, 2004). When a purposeful human action occurs it is because the particular human being whose action is being considered has chosen to so act. For example when A sells a book to B for $100, A did so with a purpose and B also did so with a purpose; they

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[14] For an alternative and more correct view of marginal utility, see (Mises, 1998, 119).

William Barnett II and Walter Block

each *chose* to act as they did. It is this essential difference between economics, on the one hand, and mathematics and physics on the other that makes mathematics inappropriate as a means of economic theory and analysis. However, because mathematics has proven immensely fruitful for physics, the vast majority of economists have adopted it as a, if not the primary, method of economics in the hope of emulating the success of physics. The answer, then, is "physics envy" (Prowse, 1996).

II. Singularism

Yet another difficulty with the demand curve is that it is logically incompatible with the "singularistic" insight of Mises (1998).[15] Here, there is one and *only* one choice enjoyed by the economic actor: to pick or to set aside (Barnett and Block, unpublished). That is, a man is *always* faced with, and limited to, the primordial option of this or that; of picking A, or setting it aside and choosing, instead, B. There are no exceptions to this rule. Any seeming exemption appears so only for lack of further or deeper analysis.[16, 17]

The demand curve is a perfect case in point. At first glance it appears to offer far more choices than merely two. It is drawn with a myriad of options implicit in it, indeed, an infinite number of them. At every possible price we are asked to mention a hypothetical amount of the good or service we would purchase. But in a real market, there can be only one price with which we are confronted. The choice is, then, to pay the price and acquire the object, or not pay the price and not acquire it.

A critic might reply something to the effect that the demand curve is logically consistent with the binary perspective in that at any given price the consumer is asked, in effect, a simple binary question: "Do you prefer one (more) unit of the good in question, or do you rather wish to keep the money necessary for its purchase in your own pocket?" And, based upon how the demand curve is drawn, at any point on it there is a clear preference indicated: the consumer preferred to purchase the item at

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[15] Mises (1998, 44), in Ch. II, §2, "The Principle of Methodological Singularism" states: "No less than from the action of an individual praxeology begins its investigations from the individual action" Mises (1998, 44). And, "Human life is an unceasing sequence of single actions" Mises (1998, 45).

[16] For example, there might be a choice between one apple and not one but two bananas, A and B, or if you prefer, #1 and #2. In this case the problem would be lack of clear specification. Either A and B constitute a single package or they do not. Is the choice between the apple and a "package" of the two bananas, A and B? Or, is it a choice either between the apple and one banana; i.e., between the apple and banana A, or between the apple and banana B? In the former case, the choice is between the apple and the package consisting of the two bananas, A and B. In the latter case, the choice is either between the apple and banana A or between the apple and banana B, as the decision maker thinks relevant. Regardless of which it is, there is still only a single choice between two options.

[17] To borrow leaves from other disciplines, although it may not appear so, biologists tell us that all living matter is composed of cells, and in the view of chemists, what matters is the numbers of neutrons and electrons, as different isotopes of the same element have the same chemical properties. The naked eye sees none of this, and thus is understandably suspicious of these claims. In like manner, while it would appear that our choices are practically limitless, every human action consists of a single choice between two possibilities.

William Barnett II and Walter Block

the given price, rather than keep his money and do without it. But this will not do, as it necessarily implies that the purchaser has more than one choice.

These insights, however correct, fail to reckon with the fact that the demand curve is no more and no less than the geometric representation of a functional equation. And, for reasons given in the previous section, this type of mathematical formulation is incompatible with choosing and setting aside.

IV. Other problems

The demand curve is usually drawn as a continuous line. This implies that there are an infinite number of prices at which a choice can be made as to how much of a given good to purchase, and thus is hardly compatible with human action which is discrete and discontinuous. Here we have a mathematical tail wagging an economic dog. When mathematics is utilized in economics, it is the former that is supposed to help shed light on the latter, not the inverse. Mathematics is the means, economics the end. But, in this case, the smooth curve assumption has no foundation in economics, in real world commercial activity. However, it is crucial for mathematics, for without it there are no differentials, derivatives, etc.

The area lying under the demand curve is interpreted in mainstream economics as the value placed upon purchasing a given amount of the product depicted on the x-axis. This, too, is logically incompatible with realistic depictions of economics, which consist of preferring and setting aside.

This practice is nonsensical from an economic point of view. Consider the concept of the market demand curve. What is relevant is human action, so the only thing that counts is the plan of the individual decision maker. If we are talking about an entrepreneur, or his agent with authority to set the price for a given good, then, ex ante, he may be considering a range of prices. It will not be a continuous range; i.e., he will not consider every possible price in that range.[18] Even if he did, it would not meet the mathematical conditions for continuity, in turn a necessity for differentiability, for there is no reason to think that, even if the domain is continuous, the range must be. So the idea of using differential or integral calculus involving demand curves and the functions they graph, as well as attendant total revenue and marginal revenue curves, is antimathematical. Moreover, although there are some instances in which the seller is concerned with the preferences of each individual potential purchaser and, therefore, is interested in each one's "demand curve," in such cases they are not interested in the "market demand curve," as their aim is differential pricing;[19] e.g., automobile retailers. However, in general, mass retailers; e.g., Wal-Mart, do not engage in differential pricing, and though they are interested in the

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[18] This is obvious for most items; e.g., gasoline retailers in the U.S. consider only prices that of that have three digits to the right of the decimal point, the last of which is always a nine. A seller of men's shirts looking at the range $50 - $60, does not seriously contemplate $50.00, $50.01, $50.02, ….. $59.98, $59.99, $60.00.

[19] The standard term for this is "price discrimination." It is true that discrimination is a perfectly good English word; e.g., it is very informative to say of someone re music, "He has discriminating tastes – he prefers Beethoven to Rock." Unfortunately, in the context of pricing, "discrimination" has acquired a pejorative connotation, and has even been criminalized in some circumstances. Therefore, we use the more neutral term, "price differential(s)."

William Barnett II and Walter Block

market demand, do not arrive at it by "horizontally summing" the individual demand curves of their potential customers.

Then there is the perspective of the potential purchaser. For the individual, the market demand is irrelevant, save as it is manifested to him in terms of the prices he faces for a particular good in the market; he does not sum up, whether horizontally or otherwise, individual demands, including his own, to get to "the" market demand. Moreover, if the good in question is of a nature that the individual is only concerned with the occasional purchase of a single unit; e.g. a house, car, or refrigerator, then although one could conceive of an individual's demand function, the only real issue for the buyer is his reservation price − reservation price in the sense that at any price above this, he reserves his money and does not buy the good; i.e., it is a buyer's reservation price that is the maximum he will freely pay, just as the seller's reservation price is the minimum he will voluntarily accept.

Perhaps most important is to see that it is sellers who are concerned with demand, and buyers with supply.

Now consider summing areas under demand curves. As there are no demand curves in reality, there is no way to add up the areas under them. Thus the concept of "consumers' surplus" as measured that way violates fundamental methodological principles of economics. Because values are only manifested in action, and all we can know is that at $5, Mary bought three units today and Charles bought two, so that a total of 5 units were sold at $5. There is no way of knowing how much an either individual would have bought at any price greater than $5, so there is no possible way to calculate the consumers' surplus.

V. Conclusion

Mises (1998, 333) offered these words of wisdom on this subject:

It is possible to visualize this interaction by drawing two curves, the demand curve and the supply curve, whose intersection shows the price. It is no less possible to express it in mathematical symbols. But it is necessary to comprehend that such pictorial or mathematical modes of representation do not affect the essence of our interpretation and that they do not add a whit to our insight. Furthermore it is important to realize that we do not have any knowledge or experience concerning the shape of such curves. Always, what we know is only market prices--that is, not the curves but only a point which we interpret as the intersection of two hypothetical curves. The drawing of such curves may prove expedient in visualizing the problems for undergraduates. For the real tasks of catallactics they are mere byplay.

To which we add, the soi-disant graphs of demand curves are antimathematical.

Should then they be abandoned? As they have great pedagogical value they should not be discarded. However, they should be treated as no more than metaphors. And, no attempt should be made to develop them as if they are mathematically valid and, therefore, add rigor to economic theory and analysis. Nor should the "functions"

William Barnett II and Walter Block

they graph be manipulated mathematically, as if such "functions" are mathematically valid and such operations yield incontrovertible economic truths.

References:

Anderson, William. 2002. "Mathematics and Economic Analysis." April 02. http://www.mises.org/story/926

Anderson, William. 2001. "The Limits of Mathematics." November 29 http://www.mises.org/story/839

Barnett, William II. 2003. "The Modern Theory of Consumer Behavior: Ordinal or Cardinal?" *Quarterly Journal of Austrian Economics.* 6 (1): 41-65. http://www.mises.org/journals/qjae/pdf/qjae6_1_3.pdf

Barnett II, W. 2004. "Dimensions and Economics: Some Problems." *Quarterly Journal of Austrian Economics.* Spring. Vol. 7, No. 1, pp. 95-104 http://www.mises.org/journals/qjae/pdf/qjae6_3_2.pdf http://www.mises.org/journals/qjae/pdf/qjae7_1_10.pdf

Barnett, William and Walter Block. Unpublished. "Singularism: Human Action is Binary."

Block, Walter. 1999. "Austrian Theorizing, Recalling the Foundations: Reply to Caplan." *Quarterly Journal of Austrian Economics*, Vol. 2, No. 4, winter, pp. 21-39; http://www.mises.org/journals/qjae/pdf/qjae2_4_2.pdf; errata: http://www.mises.org/journals/qjae/pdf/qjae2_4_9.pdf

Block, Walter. 2003. "Realism: Austrian vs. Neoclassical Economics, Reply to Caplan." *Quarterly Journal of Austrian Economics*, Vol. 6, No. 3, Fall, pp. 63-76; http://www.mises.org/journals/qjae/pdf/qjae6_3_4.pdf

Herbener, Jeffrey M. 1996. "Calculation and the Question of Arithmetic." *Review of Austrian Economics*, 9(1), pp. 151-162. http://www.mises.org/journals/rae/pdf/rae9_1_9.pdf

Hülsmann, Jörg Guido. 1999. "Economic Science and Neoclassicism." *Quarterly Journal of Austrian Economics*, Vol. 2 Num. 4, pp. 1-20; http://www.mises.org/journals/qjae/pdf/qjae2_4_1.pdf

Leoni, Bruno, and Frola, Eugenio. 1977. "On mathematical thinking in economics," *The Journal of Libertarian Studies*, Vol. 1, No. 2, Spring, pp. 101-110. http://www.mises.org/journals/jls/1_2/1_2_3.pdf

Mises, Ludwig von. 1977. "Comments about the mathematical treatment of economic problems," *The Journal of Libertarian Studies*, Vol. 1, No. 2, Spring, pp. 97-100. http://www.mises.org/journals/jls/1_2/1_2_2.pdf

Mises, Ludwig von. [1949] 1998. *Human Action,* Scholars' Edition. Auburn: Mises Institute. http://www.mises.org/humanaction.asp

Prowse, Michael. 1996. "Austrian Economics and the Public Mind." *Austrian Economics Newsletter.* Spring, Vol. 16, No. 1. http://www.mises.org/journals/aen/aen16_1_1.asp

Rothbard, Murray N. 2004. *Man, Economy and State with Power and Market.* Scholars ed. Ludwig von Mises Institute. http://www.mises.org/rothbard/mespm.PDF

Rothbard, Murray N. 1988. "Chaos Theory: Destroying Mathematical Economics from Within?" The Free Market. Vol. VI, No. 3. http://www.mises.org/freemarket_detail.asp?control=296

Shostak, Frank. 2002. "What is Wrong with econometrics?" April 17. http://www.mises.org/story/940

William Barnett II and Walter Block

Stigler, George J and Becker, Gary S. 1977. "De Gustibus Non Est Disputandum." American Economic Review. 67(2); 76-90. http://www.jstor.org/view/00028282/di950131/95p0253l/0?currentResult=00028282%2bdi950131%2b95p0253l%2b0%2c00&searchUrl=http%3A%2F%2Fwww.jstor.org%2Fsearch%2FAdvancedResults%3Fhp%3D25%26si%3D1%26All%3Dde%2Bgustibus%26Exact%3D%26One%3D%26None%3D%26ti%3Don%26sd%3D%26ed%3D%26jt%3D%26ic%3D00028282%7C15325059%7C10497498%26node.Economics%3D1%26node.Education%3D1

West, Edwin, and Michael McKee. 1983. "De Gustibus Est Disputandum: The Phenomenon of "Merit Wants" Revisited." *The American Economic Review.* 73 (5): 110-1121. http://www.jstor.org/view/00028282/di950035/95p0018k/0?currentResult=00028282%2bdi950035%2b95p0018k%2b0%2c00&searchUrl=http%3A%2F%2Fwww.jstor.org%2Fsearch%2FAdvancedResults%3Fhp%3D25%26si%3D1%26All%3Dde%2Bgustibus%26Exact%3D%26One%3D%26None%3D%26ti%3Don%26sd%3D%26ed%3D%26jt%3D%26ic%3D00028282%7C15325059%7C10497498%26node.Economics%3D1%26node.Education%3D1

Wutscher, Robert. Unpublished. "Foundations in economic methodology: The use of mathematics by mainstream economics and its methodology by Austrian economics." http://www.mises.org/journals/scholar/Wutscher.pdf