MODELS FOR MAKING DECISIONS ON THE VOLUME AND PRICE OF PURCHASES IN WHOLESALE TRADE

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Abstract: The article proposes a mathematical model to support decision-making on the volume of purchases and the level of retail prices, which the wholesale company (dealer) sets depending on the manufacturer's selling prices and the demand of end buyers for various volumes of wholesale purchases. It is assumed that the manufacturer and the wholesaler influence price through pricing policy. As a result, the problem of determining the optimal volume of wholesale purchases and sales, depending on supply and demand, as well as changes in the level of purchase and selling prices, is posed and solved. Two options for setting the problem are considered here: when the end buyer is informed and not informed about the producer's prices.

Keywords: wholesale trade, mathematical modeling, optimization, pricing, seller, dealer, buyer.

JEL: F14, F17.

Introduction

Wholesale trade has an important place in the formation of sectoral and regional commodity markets, acting as an organizer of the market and market relations. A wholesale company, being both a buyer and a seller, is an economic entity that is an intermediate link in the chain between product

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manufacturers and business entities or individuals who purchase goods. The main difference between wholesale and retail trade is that wholesale companies, regardless of volumes, do not sell to the final consumer, but to the next reseller. It means that this type of business relationship falls into the B2B category, and we know that the success of such a business depends heavily on the quality of decision-making expertise and scientific validity. In this regard, the introduction of statistical and economic-mathematical methods, artificial intelligence and machine learning, and general and specialized software into the management practice of such companies is of great importance.

The most serious university programs for the preparation of management for trade organizations include, as a mandatory component, teaching the mathematical skills necessary for success in business, and mastering the business concepts that require mathematical solutions. At the same time, special attention is paid to the practical aspects of applying the entire range of basic business mathematics in solving real problems of operating companies, according to various business scenarios (Carman, Saunders, 1986; Clendenen, Salzman, 2014). This time-tested pedagogical practice has made it possible to prepare a huge number of highly qualified specialists for trading companies capable of effectively using the economic and mathematical apparatus offered by the scientific and professional community.

The main interest of a wholesale trading company is to increase the efficiency of its business. A large variety of ways, methods and approaches are theoretically capable of increasing the efficiency of a company, but they are not universally applicable. The application of the same theory in one case allows the user to increase income, but in another case, it is practically useless. This explains why a myriad of approaches are welcomed, reflecting the various aspects of this complex and diversified business (Kingsnorth, 2019; Saunders, Emeritus, 2018).

A wide range of issues related to improving the efficiency of commercial enterprises, including macro- and microeconomic aspects, lies in the field of scientific interests of mathematicians and economists. Some publications present not only specific decision-making tools and frameworks, but also offer guidance on how to apply them to large and small firms. The authors of the works, based on real entrepreneurial and managerial experience, as well as advanced scientific research, present various options for marketing decisions from around the world in companies from small start-ups to international giants, in the context of numerous socio-economic phenomena (Iliev, Stefanov & Yotov, 2016; Alekseeva, 2017).

The problem of pricing is the most relevant for trade organizations, and it is of particular importance for companies operating in highly competitive markets with many actors offering similar or identical goods and services. The competitive struggle for the consumer in this situation is resolute and uncompromising. Under these conditions, old marketing methods and techniques do not often work or are minimally effective, which makes it necessary to search for new marketing solutions and their theoretical and empirical justification (Petrova, 2021).

This article discusses the specific tasks of increasing the profitability of a trading business in a highly competitive environment through science-based decision-making on the volume of purchases and the value of retail prices that a wholesale company (dealer) sets, based on changes in the manufacturer's selling prices and end-customer demand.

At the same time, it is assumed that the manufacturer pursues an active pricing policy on the market, stimulating demand by reducing the selling price. In turn, the wholesale company can also significantly influence pricing by determining the volume of purchases and sales of goods. This provision leads to the fact that all market participants, with the exception of end consumers, are faced with the task of determining the optimal volumes of wholesale purchases and/or sales, depending on supply and demand, as well as changes in the level of purchase and selling prices.

Next, we consider two options for setting and solving this problem in order to maximize the profit received by a reseller – a wholesale trading company or a dealer.

The Uninformed Buyer Model

When constructing the first model, we will proceed from the following assumptions:

I. There is a three-level structure of market participants:

a) the manufacturer;

b) wholesaler (dealer);

c) buyers.

Their interaction is implemented as follows:

The dealer purchases goods directly from the manufacturer at selling prices. Buyers do not directly interact with the manufacturer and buy goods only through a dealer. We will call such a buyer "uninformed".

II. The dependence of the total consumer demand $\boldsymbol{V}$ on the dealer selling price per unit $\boldsymbol{q}$ is determined by the function $\boldsymbol{V(q)}$. When constructing the model, we will proceed from the assumption that this function is linear.

The choice of a linear relationship is explained by the assumption that the manager of a dealership can determine the following values with a sufficient degree of accuracy:

$\boldsymbol{b_1}$ – the minimum wholesale selling price of the manufacturer, at which the quantity of goods ($\boldsymbol{V_0}$) can be sold;

$\boldsymbol{b_2}$ – the price of the product at which no buyer will buy it.

The dependence of demand on price $\boldsymbol{V(q)}$ can be built on the basis of these data.

III. The dependence $\boldsymbol{b(V)}$ of the manufacturer's shipping prices (b) on the volume of dealer purchases $\boldsymbol{(V)}$ is known. This dependence can be specified as a step function decreasing in $\boldsymbol{V}$. The maximum value of the price $b(V)$ is b_0, which occurs when purchasing one unit of product, and the minimum is $\boldsymbol{b_1}$ at the minimum shipping price of the manufacturer, determined by the amount of variable costs spent by the manufacturer for the production of one product.

In this model, the dependence of the price $\boldsymbol{(b)}$ on the volume of purchases $\boldsymbol{V}$ for a certain period (for example, within a year) can be approximated by a straight broken line consisting of two sections:

a) decreasing from the point with coordinates $\boldsymbol{(V = 1,\ b = b_0)}$, which corresponds to the manufacturer's shipping price when purchasing 1 unit of goods (the minimum reasonable quantity of the manufacturer's products from the point of view of the dealer's fixed costs), to the point with coordinates $\boldsymbol{(V = V_1,\ b = b_1)}$ corresponding to the volume of purchases $\boldsymbol{V_1}$, starting from which the manufacturer sells the goods to the dealer at the lowest possible price $\boldsymbol{b_1}$.

b) parallel to the axis $\boldsymbol{V}$ at $\boldsymbol{b = b_1}$, which corresponds to any sales volume of the manufacturer to the dealer, starting from $\boldsymbol{V_1}$ (if to be absolutely precise, starting from $\boldsymbol{V_1 + 1}$). The sale is made at the price $\boldsymbol{b = b_1}$.

Figure 1 shows the dependences ***V (q)*** and ***b (V)*** for the following parameter values: V_1 = 51, V_0 = 100, b_0 = 1.0, b_1 = 0.5, b_2 = 2.0.

Let's formulate the problem:

It is necessary to determine the amount of wholesale purchases from the manufacturer made by the dealer – *Vmax*, calculated in value or in kind, which ensures the maximization of the dealer's profit, calculated by the formula *P = (q - b) V*, where *q* is the selling price at which the seller sells goods to the buyer, *b* is the manufacturer's selling price with the volume of purchases *V*, *V* is the volume of wholesale purchases.

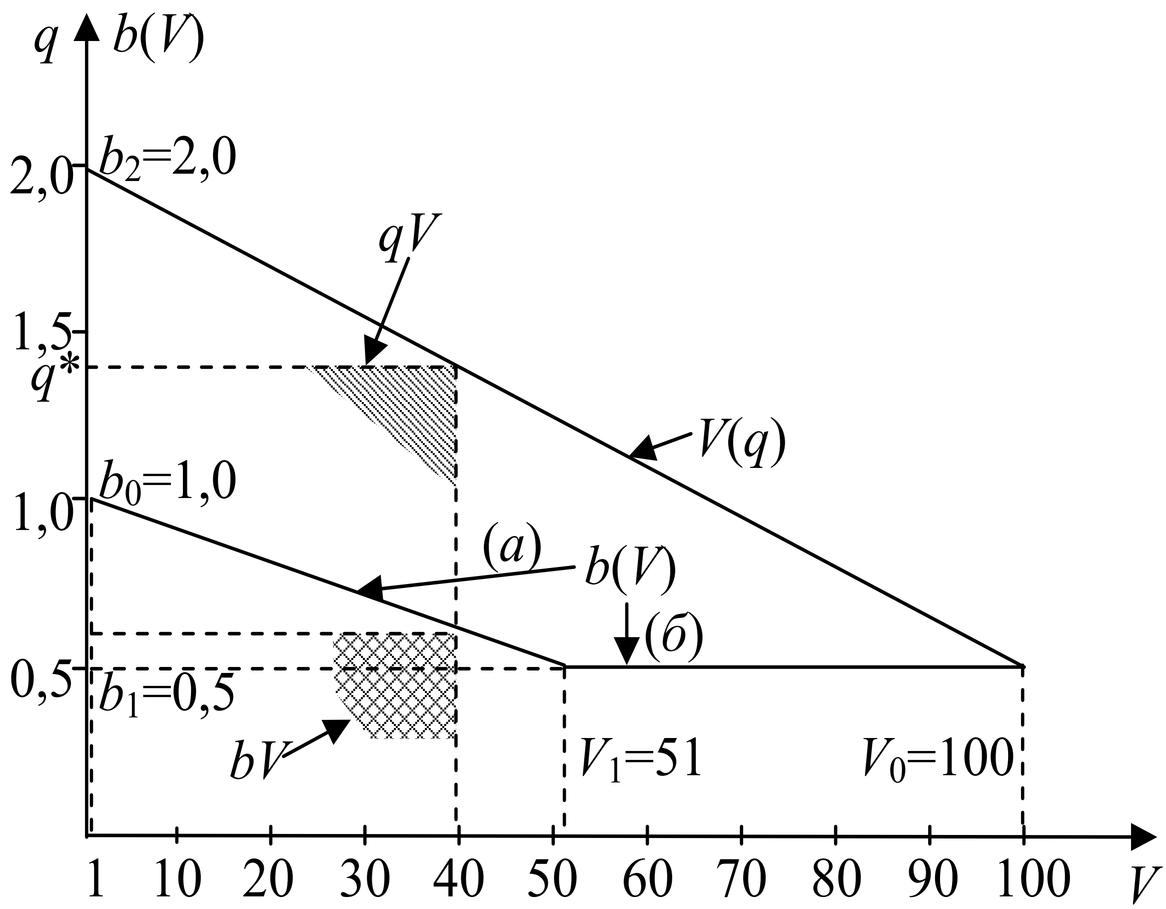

Figure 1. Graphical representation of the decision-making model on the volume of purchases.

That is, it is necessary to solve the following optimization problem:

$$b(V) =$$
$$= \begin{cases} (b_1 - b_0 V_1)/(1 - V_1) + [(b_0 - b_1)/(1 - V_1)] \ \ V, 1 \le V \le V_1 & (a) \\ b_1, V > V_1 & (б) \end{cases} \quad (1)$$

Curve *V(q)* is described by the relation

$$P = \left(q(V) - b(V)\right)\ V \to \max_{V}.$$

Next, consider the solution to this problem.

Based on the above, the *b(V)* curve is described by the following relationships:

$$b(V) = \\ = \begin{cases} (b_1 - b_0V_1)/(1 - V_1) + [(b_0 - b_1)/(1 - V_1)]\ \ V, 1 \le V \le V_1 & (a) \\ b_1, V > V_1 & (б) \end{cases} \qquad (1)$$

Curve *V(q)* is described by the relation

$$V(q) = \frac{V_0 b_2}{b_2 - b_1} + \left[\frac{V_0}{b_1 - b_2}\right]\ \ q.$$

Let's solve this relation for *q* relative to *V*:

$$q = b_2 - [(b_2 - b_1)\ V_0]\ V.$$

It is required to determine the value of *V,* at which the function *P* on *V* reaches a maximum:

$$P(V) = \left(q(V) - b(V)\right)\ V \to \max_{V}.$$

First, we solve this problem for the case when the function *b(V)* has the form *b(V) = b1* (case (*b*)). Then

$$P(V) = \left(b_2 - \frac{b_2 - b_1}{V_0} V - b_1\right) V, \quad \frac{dP}{dV} = b_2 - b_1 - \frac{2(b_2 - b_1)}{V_0} V = 0,$$

and then $V = V_0 / 2$.

Since $\left(\frac{dP}{dV}\right)'' = -2\frac{b_2 - b_1}{V_0} < 0$, and from the meaning of the problem $b_2 > b_1$, then the function P(V) has a maximum at the point $V=V_0/2$.

If $V_0/2 > V1$, then the maximum point at $V = V_0/2$ is valid; if not, then it is not valid.

Let us now solve the problem for the case when the function *b(V)* has the form (a), then:

$$P(V) = \left\{b_2 - \frac{b_2 - b_1}{V_0} V - \frac{b_1 - b_0 V_1}{1 - V_1} + \frac{b_0 - b_1}{1 - V_1} V\right\} V$$

$$\frac{dP}{dV} = b_2 - 2\frac{b_2 - b_1}{V_0} V - \frac{b_1 - b_0 V_1}{1 - V_1} + 2\frac{b_0 - b_1}{1 - V_1} V,$$

and

$$V = V^* = \frac{V_0}{2} \times \frac{(b_2 - b_0)\ \ V_1 + b_1 - b_2}{(b_2 - b_1)(V_1 - 1) + V_0(b_0 - b_1)}$$

$$\left(\frac{dP}{dV}\right)'' = 2\frac{b_0 - b_1}{1 - V_1} - 2\frac{b_2 - b_1}{V_0}.$$

Since it follows from the meaning of the problem that $b_0 > b_1$, $b_2 > b_1$, $V_1 > 1$, then $(dP/dV)'' < 0$ and the value $V = V^*$ is the maximum of the function $P(V)$.

If V^* satisfies the inequality $V^* <= V_1$, then the maximum point at $V=V^*$ is valid; if not, then it is not valid.

If both maximum points, at $V = V_0/2$ and at $V = V^*$, are admissible, then to solve the problem $P(V) \underset{V}{\rightarrow} max$, it is necessary to compare the values of the objective function $P(V)$ at $V = V_0/2$ and for $V = V^*$. That value of V, for which $P(V)$ will be greater, will be the maximum point.

Thus, the value $Vmax = arg\ max\{P(V_0/2), P(V^*)\}$ will be the value at which the dealer purchases from the manufacturer, which will provide the dealer with maximum profit.

The Informed Buyer Model

Let's consider the second problem, which we will call the "informed buyer model".

In this variant, when constructing the model, we will proceed from the following assumptions:

I. The buyer is aware of:

1. Selling prices of the manufacturer **b** and their dependence on the volume of wholesale purchases ***V***.
2. The volume of wholesale purchases of the seller (dealer) ***V***.

The buyer who has such information will be called "informed", and the model built for this case will be called the informed buyer model.

II. Assume that an informed buyer considers a normal price $q = (1+k)b(V)$, which is $kb(V)$ higher than the manufacturer's selling price $b(V)$ and includes all the seller's (dealer's) fixed and variable costs. Demand V on the part of buyers for a product sold by a dealer at a price $q = (1+k)\ b(V)$ is equal to the demand for a product sold by a manufacturer at a price $b(V)$. At the same time, we also proceed from the fact that buyers cannot purchase goods directly from the manufacturer.

III. When the selling price of the seller q is greater than $(1 + k)b(V)$, with the volume of wholesale purchases V, the demand W for products from the buyers begins to fall. Buyers, for example, can switch their demand to products from other manufacturers.

Within this model, it is necessary to determine the volume of bulk purchases *V** and the selling price *q**, which will provide the dealer with maximum profit.

Managers of the selling firm (dealer) can estimate the value *m = q/b(V),* which corresponds to the value of the price q, at which the demand *W* for products purchased in quantity *V* at price *b(V)* and sold by the dealer at price *q*, will be equal to zero.

Since at the price value *q = (1+k) b(V),* the demand W from buyers for the products purchased by the dealer in volume *V* is equal to *V*, and at the price *q = mb(V)*, it is equal to 0, then we can determine the coefficients of the linear dependence *W(q),* the parameters of which are the values *V* and *b(V).*

The dealer's managers can estimate the value *m = q/b(V),* which corresponds to the price q, at which the demand W for goods bought in quantity *V* at price *b(V)* and sold by the dealer at price q will be equal to zero. Since at the price value *q = (1+k)b(V)* the demand W from buyers for the products purchased by the dealer in volume *V* is equal to *V*, and at the price *q = mb(V)* it is equal to zero, then we can determine the coefficients linear dependence *W(q)*, whose parameters are the values of *V* and *b(V)*.

Let us determine the coefficients *a* and *c* of the straight line *W = aq + c* passing through two points on the coordinate plane *(q, W)* with coordinates *(mb(V), 0)* and *((1+k) b(V), V)*. We get:

$$a = \frac{V}{b(V)(1+k-m)}; c = \frac{Vm}{m-1-k}.$$

where

$$W = \left\{\frac{mb(V)-q}{b(V)(m-1-k)}\right\} V.$$

Based on the foregoing as a criterion, we will use the maximum profit given by the ratio:

$$P(q,V) = qW(q,V) - b(V) \;\; V \to \max_{q,V}. \qquad (2)$$

The solution of the problem.

At first, with a fixed *V (V=V*)* and, accordingly, *b(V)*, we need to consider a criterion of the form:

$$P(q,V) = qW(q,W) \to \max_{q}. \qquad (3)$$

We have

$$P(q,W) = qW = q\left\{\frac{mb(V)-q}{b(V)(m-1-k)}\right\} V \qquad (4)$$

$$\frac{dP}{dq} = \frac{mb(V)-2q}{b(V)(m-1-k)} = 0,$$

from which follows

$$q^* = mb(V)/2 \qquad (5)$$

Let's define the sign of the second derivative:

$$\left(\frac{dP}{dq}\right)'' = -2\frac{V}{b(V)(m-1-k)}.$$

Since, according to the meaning of the problem, $m - k - 1 > 0$, then

$$\left(\frac{dP}{dq}\right)'' < 0,$$

and at the point $q = q^*$ the maximum of the function $P(q,W)$ is reached. We substitute the expression for q^* from (5) into (4) and obtain an expression for W (W^*) corresponding to the maximum of the function $P(q,W)$:

$$W^* = W(q^*) = \frac{mV}{2(m-1-k)}.$$

Consider now criterion (2):

$$P(q,V) = qW(q,V) - b(V)V \to \max_{q,V}.$$

We substitute the found value $q = q^*$ into it, we get:

$$P(V) = \frac{mb(V)}{2} \times \frac{mV}{2(m-1-k)} - b(V)V = \left[\frac{m^2}{4(m-1-k)} - 1\right] b(V)V \to \max_V.$$

The function $b(V)$, as before, is given by relations (1):

$$b(V) = \begin{cases} \dfrac{b_1 - b_0V_1}{1 - V_1} + \dfrac{b_0 - b_1}{1 - V_1}V, 1 \le V \le V_1 & (a) \\ b_1, V > V_1 & (б) \end{cases}.$$

At first consider case (*b*).

In this case, the function $P(V)$ is linear in V and has the following form:

$$P(V) = \left[\frac{m^2}{4(m-1-k)} - 1\right] b_1V.$$

The maximum $P(V)$ is reached at the maximum possible value of V, in our case at $V = V_0$.

$$P(V_0) = \left[\frac{m^2}{4(m-1-k)} - 1\right] b_1V_0.$$

Now consider case (*a*):

$$P(V) = \left[\frac{m^2}{4(m-1-k)} - 1\right]\left\{\frac{b_1 - b_0V_1}{1 - V_1} + \frac{b_0 - b_1}{1 - V_1}V\right\}V,$$

$$\frac{dP}{dV} = \left[\frac{m^2}{4(m-1-k)} - 1\right]\left\{\frac{b_1-b_0V_1}{1-V_1} + 2\frac{b_0-b_1}{1-V_1}V\right\} = 0,$$

from which follows

$$V^* = \frac{b_0 V_1 - b_1}{2(b_0 - b_1)}.$$

To find out whether the maximum or minimum of the function *P(V)* is reached at the point $V = V^*$, it is necessary to determine the sign of the second derivative of *P(V)* with respect to *V* at the point $V = V^*$:

$$\left(\frac{dP}{dV}\right)'' = 2\left[\frac{m^2}{4(m-1-k)} - 1\right]\frac{b_0 - b_1}{1 - V_1} = 0.$$

Since $b_0 > b_1$ and $V_1 > 1$ by the meaning of the problem, the factor

$$\frac{b_0 - b_1}{1 - V_1} < 0$$

and to find out the sign of the second derivative, it is necessary to determine the sign of the factor

$$\frac{m^2}{4(m-1-k)} - 1:$$

if it is greater than zero, then at the point $V = V^*$ the maximum of the function *P(V)* is reached, otherwise it is not.
It should be noted that for the above values $m = 3.0$ and $k = 0.5$ this factor is positive.

Let us assume that the maximum of the function *P(V)* is reached at the point $V = V^*$; then, to solve the problem ("informed buyer"), it is necessary to compare the values of the criterion *P(V)* at $V = V_0$ and $V = V^*$: $P(V_0)$ and $P(V^*)$. The value of *V*, at which the criterion *P* will have a greater value, will be the solution to the problem.

The value q^*, determined by relation (5), and the value $V_{max} = \arg\max\{P(V_0), P(V^*)\}$ are, respectively, the value of the dealer's selling price and the volume of dealer purchases that maximize the dealer's profit and are, respectively, solving the problem that was defined above as the "informed buyer model".

Summary

In this paper, we considered two decision-making models on the volume of purchases and on the level of selling prices, which are set by the wholesale buyer depending on changes in producer prices and end-customer demand for various volumes of wholesale purchases. The main substantive difference between these models from each other is the consideration of the different degree of awareness of the buyer about the dynamics of producer prices, as a result of which two models were built for "uninformed" and "informed" buyers.

These models can be useful to financial department management and employees of wholesale companies in making decisions on the volume of wholesale purchases and the level of selling prices.

All the necessary initial numerical data used in these models are fully presented in the information bases of the accounting and analytical systems of trade enterprises. This makes it possible for the software implementation of the proposed algorithms in the environment of the relevant information systems. Possible options for the implementation of economic and mathematical models and methods in the environment of existing information systems are studied in sufficient detail by the authors of this article in their book (Zalozhnev, Chistov & Shuremov, 2022).

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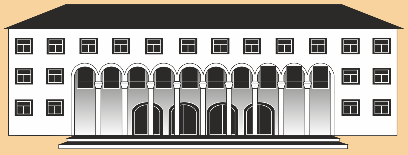

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